# Testing fundamental principles of GR with an eye on QG

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# Preface

- Some aspects of Gravity are surprisingly poorly understood. This certainly concerns its relation to Quantum(Field)Theory, but also far more down-to-earth issues.
- For example, the gravitational constant is relatively uncertain

$$G^{\mathrm{obs}} = (6.67259 \pm 0.00085) \times 10^{-11}\ \mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}$$

- As stressed by Damour (1992), this fact may leave us in awkward situations when confronted with results of speculations, like

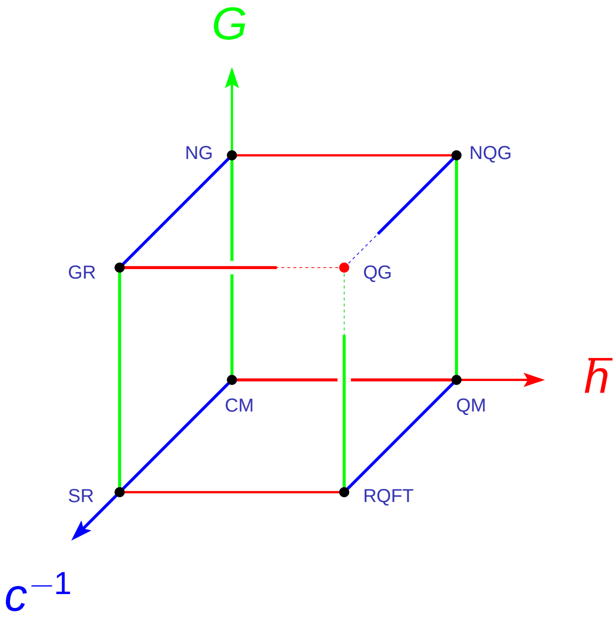
$$G^{\mathrm{theory}} = \frac{\hbar c}{m_e^2} \cdot \frac{(7\pi)^2}{5} \cdot \exp(-\pi/4\alpha)$$

  for which

$$\frac{G^{\mathrm{obs}}}{G^{\mathrm{theory}}} = 1.00004 \pm 0.00013$$

- **I take this as a warning that we should try harder to also understand our down-to-earth issues.**

# Three fundamental deformations



# Hertz’ “Die Constitution der Materie” of 1884

Heinrich Hertz (1857-94)

“But, in reality, we have two principal properties of matter before us [inertial and gravitational mass], which can be thought entirely independently of each other, and which yet prove identical by experience, and only by experience. This coincidence is a miracle and calls out for an explanation. We may conjecture that a simple explanation exists and that this explanation will gives us far reaching insights into the constitution of matter.”

# Einstein's Equivalence Principle (EEP)

- **Universality of Free Fall (UFF)** Requires existence of sufficiently general "test bodies" to determine a path structure on spacetime (not necessarily of pseudo Riemannian type). Possible violations of UFF are parametrised by the Eötvös factor

$$\eta(A,B) := 2\,\frac{|a(A)-a(B)|}{|a(A)+a(B)|} \approx \sum_{\alpha} \eta_\alpha \left( \frac{E_\alpha(A)}{m_i(A)c^2} - \frac{E_\alpha(B)}{m_i(B)c^2} \right)$$

- **Local Lorentz Invariance (LLI)** Local non-gravitational experiments exhibit no preferred directions in spacetime, neither timelike nor spacelike. Possible violations of LLI concern, e.g., variations in $\Delta c/c$.
- **Universality of Gravitational Redshift (UGR)** Requires existence of sufficiently general "standard clocks" whose rates are universally affected by the gravitational field. Possible violations of UFF are parametrised by the $\alpha$-factor

$$\frac{\Delta\nu}{\nu} = (1+\alpha)\frac{\Delta U}{c^2}$$

$\Rightarrow$ **Geometrisation of gravity and unification with inertial structure. Far reaching consequences.**

## Levels of verification of EEP

- **UFF:** Typical results from torsion-balance experiments by the "Eöt-Wash" group between 1994-2008 are

  $$\eta(Al,Pt) = (-0.3 \pm 0.9) \times 10^{-12}\,, \qquad \eta(Be,Ti) = (0.3 \pm 1.8) \times 10^{-13}$$

  Planned improved levels are $10^{-15}$ (MICROSCOPE) and $10^{-18}$ (STEP).
- **LLI:** Currently best Michelson-Morley type experiments give (Herrmann *et al.* 2005)

  $$\frac{\Delta c}{c} < 3 \cdot 10^{-16}$$

  Hughes-Drever type experiments $10^{-22}$, cosmic rays $5 \cdot 10^{-23}$ (Coleman & Glashow 1997).
- **UGR:** Absolute redshift with H-maser clocks in space (1976, $h = 10\,000\,\mathrm{Km}$) and relative redshifts using precision atomic spectroscopy (2007) give

  $$\alpha_{\mathrm{abs}} < 2 \times 10^{-4} \qquad \alpha_{\mathrm{rel}} < 4 \times 10^{-6}$$

  In Feb. 2010 Müeller *et. al.* claimed improvements by $10^4$. This is presumably incorrect (see below). Long-term expectation in future space missions is to get to $10^{-10}$ level.
- NB. In Sept. 2010 Chou *et al.* report measurability of gravitational redshift on Earth for $h = 33\,\mathrm{cm}$ using $\mathrm{Al}^+$-based optical clocks ($\Delta t/t < 10^{-17}$).

# Mechanisms for violating EEP

- **UFF(UGR):** Rest-masses of particles (properties of clocks) may be space-time dependent:

$$\ddot{x}^a + \Gamma^a_{bc}\,\dot{x}^b\dot{x}^c = (g^{ab} - \dot{x}^a\dot{x}^b)\nabla_b m/m$$

$$\Rightarrow \eta(A,B) \propto \|\nabla(m_A/m_B)\|$$

  This may happen through dependence on long-ranging scalar fields, e.g., via gauge couplings (Damour & Polyakov 1994).

- **LLI:** Anomalous dispersion due to breaking and/or deformation of Poincaré symmetry. New symmetries may appear as those of certain 'ground states' in QG

$$E^2 = (pc)^2 + \sum_{n\geq 3} f_n E_p^{2-n}(pc)^n \quad \Rightarrow \quad \frac{v_{\mathrm{gr}}}{c} = 1 + \sum_{n\geq 3} \frac{n-1}{2}\, f_n \left(\frac{E}{E_p}\right)^{n-2}$$

- Metric fluctuations show up in 'coarse-grained' Hamiltonian with resolution length $\ell$ (Göklü & Lämmerzahl 2008):

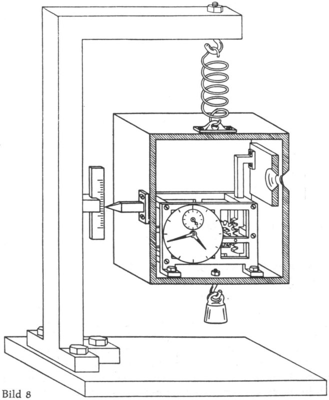
$$H_{\mathrm{kin}} = -\frac{\hbar^2}{2m}\left(\delta_{ab} + \left(\frac{\ell_p}{\ell}\right)^{\beta} A_{ab}\right)\partial_a\partial_b$$

  Values for $\beta$ correspond to various noise scenarios, e.g., 1/2 for random walk, 2/3 for holographic noise, and unity for anti-correlation.

# EP-related project involvements at ZARM Bremen

- **MICROSCOPE** (Micro Satellite à trainée compensée pour l'Observation du Principe d'Equivalence) Scheduled 2014, duration 1 year. Aim: Test UFF up to $10^{-15}$ level using extremely sensitive capacitive acceleration sensors (ONERA) on drag-free satellites allowing for long integration times.
- **QUANTUS** (Quantengase unter Schwerelosigkeit) Begun 2004, 2007 first BEC under microgravity conditions (87Rb-based). Aim is to demonstrates feasability of quantum optical experiments in such environment for later space missions.
- **PRIMUS** (Präzisionsinterferometrie mit Materiewellen unter Schwerelosigkeit). Using a BEC as matter-wave source, this is a pathfinder experiment that aims to perform first atom interferometric measurements at extended free evolution times that are available in a microgravity environment.
- **Nonlinear Schrödinger Equations**

# QM needs GR (UGR)?



- Einstein argues to be able to violate $\Delta E\,\Delta T > \hbar$.
- Bohr argues that inequality holds due to UGR:

$$\text{QM:} \qquad \Delta q > \frac{\hbar}{\Delta p} > \frac{\hbar}{Tg\,\Delta m}$$

$$\text{ART:} \qquad \Delta T = \frac{gT}{c^2}\,\Delta q$$

$$\Longrightarrow \qquad \Delta T > \frac{\hbar}{\Delta m\, c^2} = \frac{\hbar}{\Delta E}$$

- Bohr's argument is presumably not right, but its underlying logic seems remarkable.

# QFT needs GR (SEP): Gravity as regulator?

- Consider thin mass shell of Radius $R$, inertial rest-mass $M_0$, gravitational mass $M_g$, and electric charge $Q$. Its total energy is

$$E = M_0 c^2 + \frac{Q^2}{2R} - G\frac{M_g^2}{2R}$$

- Now use the following two principles:

$$\begin{aligned} E &= M_i c^2 \\ M_g &= M_i \end{aligned}$$

- Get quadratic equation for mass $M := M_i = M_g$:

$$\Rightarrow \quad M := \frac{E}{c^2} = M_0 + \frac{Q^2}{2c^2 R} - G\frac{M^2}{2c^2 R}$$

# Gravity as regulator (contd.)

- The solution is

$$M(R) = \frac{Rc^2}{G}\left\{-1 + \sqrt{1 + \frac{2G}{Rc^2}\left(M_0 + \frac{Q^2}{2c^2R}\right)}\right\}$$

- Its $R \to 0$ limit exits

$$\lim_{R\to O} M(R) = \sqrt{\frac{2Q^2}{G}} = \sqrt{2\alpha}\cdot\frac{|Q|}{e}\cdot M_{\text{Planck}}$$

but its small-G approximation is not uniform in $R$ at $R=0$:

$$\begin{aligned} M &= \left(m_0 + \frac{Q^2}{2c^2R}\right)\\ &+ \sum_{n=1}^{\infty}\frac{(2n-1)!!}{(n+1)!}\cdot\left(-\frac{G}{Rc^2}\right)^n\cdot\left(m_0+\frac{Q^2}{2c^2R}\right)^{n+1}\end{aligned}$$

# UFF in QM

- As a special case of EEP we obtain the statement, that non-gravitational physics in a homogeneous gravitational field is indistinguishable from that in a constantly accelerated frame of reference.
- Wave packets are not structureless and cannot be expected to realise test particles (except for their centre-of-mass motion due to Ehrenfest's theorem).
- What, if any, is the analogy to classical equivalence of *homogeneous* gravitational fields with constant acceleration?

# A proposition

$\psi$ solves the Schrödinger equation

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m_i}\Delta - \vec{F}(t)\cdot\vec{x}\right)\psi$$

iff

$$\psi = \bigl(\exp(i\alpha)\,\psi'\bigr)\circ\Phi^{-1}$$

where $\psi'$ solves the free Schrödinger equation (i.e. without potential). Here $\Phi:\mathbb{R}^4\to\mathbb{R}^4$ is the following spacetime diffeomorphism (preserving time)

$$\Phi(t,\vec{x}) = \bigl(t,\vec{x}+\vec{\xi}(t)\bigr)$$

where $\vec{\xi}$ is a solution to

$$\ddot{\vec{\xi}}(t) = \vec{F}(t)/m_i$$

with $\vec{\xi}(0)=\vec{0}$, and $\alpha:\mathbb{R}^4\to\mathbb{R}$ given by

$$\alpha(t,\vec{x}) = \frac{m_i}{\hbar}\left\{\dot{\vec{\xi}}(t)\cdot\bigl(\vec{x}+\vec{\xi}(t)\bigr) - \frac{1}{2}\int^t dt'\,\|\dot{\vec{\xi}}(t')\|^2\right\}$$

# UFF in QM

Application of this to time independent and homogeneous gravitational field

$$\vec{F} = -m_g g \vec{e}_z$$

shows that wave function at time $t$ is obtained from freely evolved wave function at time $t$, with same initial data, by:

- Galilean boost with classical velocity
- rigid motion along classical trajectory

Both depend only on quotient $m_g/m_i$.

**This type of rigid fall of the wave packet is the closest analog of UFF in QM one could have hoped for.**

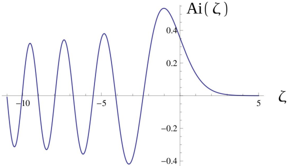
# Stationary states

- The time-independent Schrödinger equation with potential $V = m_g g z$ is equivalent to (cf. Kajari *et al.* (2010)):

$$\left(\frac{d^2}{d\zeta^2} - \zeta\right)\psi = 0\,, \qquad \zeta := \kappa z - \varepsilon$$

  where

$$\kappa := \left[\frac{2m_i\, m_g\, g}{\hbar^2}\right]^{\frac{1}{3}}\,, \qquad \varepsilon := E \cdot \left[\frac{2m_i}{m_g^2\, g^2\, \hbar^2}\right]^{\frac{1}{3}}\,.$$

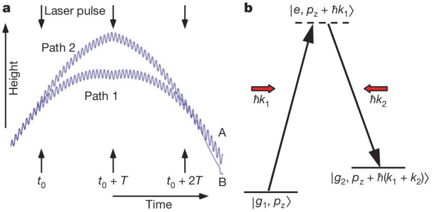


- Bounce-back (Peres-) 'time' (Davies 2004) again just depends on quotient $m_i/m_g$:

$$T_{\mathrm{ret}} = 2 \cdot \left[\frac{m_i}{m_g}\right]^{\frac{1}{2}} \cdot \left[\frac{2h}{g}\right]^{\frac{1}{2}}$$

- Stationary states of few $10^{-12}\,\mathrm{eV}$ seen (Abele *et al.* 2002, Laue-Langevin Grenoble) with ultracold neutrons in search for anomalous gravitational interaction below $10^{-5}\,\mathrm{m}$.

# Recent confusions:
# The Argument of Müller, Peters, and Chu (Nature 2010)



Atom interferometer and 2-photon Raman interaction with $\pi/2$-pulses (beam splitters) at $t = t_0$ and $t = t_0 + 2T$, and a $\pi$-pulse (mirror) at $t = t_0 + T$. Each time the total vertical momentum $\kappa = \|\vec{k}_1\| + \|\vec{k}_2\|$ is transferred.
(Müller *et al.* 2010).

$$\Delta\phi = \Delta\phi_{\text{redshift}} + \underbrace{\Delta\phi_{\text{time}} + \Delta\phi_{\text{light}}}_{=0} = \Delta\phi_{\text{redshift}}$$

$$= \underbrace{\Delta\phi_{\text{redshift}} + \Delta\phi_{\text{time}}}_{=0} + \Delta\phi_{\text{light}} = \Delta\phi_{\text{light}}$$

# Recent confusions (cont'd)

Have

$$\Delta\phi = \Delta\phi_{\mathrm{redshift}} = (1+\alpha)\kappa T^2 g'$$

where

$$g' := (m_g/m_i)\, g$$

- Hence the redshift per unit length is

$$z := (1+\alpha)\frac{g'}{c^2} = \frac{\Delta\phi}{\kappa T^2 c^2}$$

- The measured versus the predicted (taking systematic corrections into account) values are

$$\begin{aligned} z_{\mathrm{meas}} &= (1.090\,322\,683 \pm 0.000\,000\,003) \times 10^{-16}\,\mathrm{m}^{-1} \\ z_{\mathrm{pred}} &= (1.090\,322\,675 \pm 0.000\,000\,006) \times 10^{-16}\,\mathrm{m}^{-1} \end{aligned}$$

  which translates to

$$\alpha = \frac{z_{\mathrm{meas}}}{z_{\mathrm{pred}}} - 1 = (7 \pm 7) \times 10^{-9}.$$

- This should be compared to previous tests (Gravity-Probe-A, 1976) using hydrogen masers in rockets at altitude 10 000 Km $(2 \times 10^{-4})$ and planned ones (launch 2013) on the ISS (ACES, $2 \times 10^{-6}$).

# Recent confusions (cont'd)

- The authors observed that *formally* $\Delta\phi = \Delta\phi_{\text{redshift}}$, independent of whether $g' = g m_h/m_i$. Hence they thought it legitimate to replace $g' \to (1+\alpha)g'$:

$$\Delta\phi = \underbrace{\kappa T^2 g'}_{\Delta\phi_{\text{time}}} - \underbrace{\kappa T^2 (m_g/m_i)(1+\alpha)g}_{\Delta\phi_{\text{redshift}}} - \underbrace{\kappa T^2 g'}_{\Delta\phi_{\text{light}}}$$

- The unknown $g$ is eliminated through a nearby reference measurement of the acceleration $\bar{g} = (M_g/M_i)g$ of a corner cube of inertial mass $M_i$ and gravitational mass $M_g$.
- Using the Nordtvedt parameter for the atom-cube pair,

$$\eta := \eta(\text{atom}, \text{cube}) := 2\,\frac{(m_g/m_i) - (M_g/M_i)}{(m_g/m_i) + (M_g/M_i)}$$

  we get for the total phase shift:

$$\Delta\phi = -\kappa T^2 \bar{g} \quad (1+\alpha)\frac{2+\eta}{2-\eta} \approx -\kappa T^2 \bar{g} \quad (1+\alpha)(1+\eta)$$

- Back to solid ground, $\alpha = 0$, it is undisputed that this can be used as accelerometer to measure $\eta$, though not yet to the same level of precision as other tests.

# Schrödinger-Newton Equation

- Semi-classical Einstein equation

$$R_{ab} - \tfrac{1}{2} g_{ab} R = \kappa \langle T_{ab} \rangle_{\psi}$$

- Schrödinger-Newton (Choquard) equation

$$i\hbar \partial_t \Psi(t,\vec{x}) = \left( -\frac{\hbar^2}{2m} \Delta - G m^2 \int \frac{|\psi(t,\vec{y})|^2}{\|\vec{x}-\vec{y}\|} \, d^3y \right) \Psi(t,\vec{x}) = 0$$

- Introduce length scale, *a*, and use dimensionless variables:

$$\vec{x}\,' := \vec{x}/a\,, \quad t\,' := t\hbar/(2ma^2)\,, \quad \Psi' := a^{3/2}\Psi$$

  and get

$$i\partial_{t'} \Psi'(t',\vec{x}\,') = \left( -\Delta' - C \int \frac{|\psi(t',\vec{y}\,')|^2}{\|\vec{x}\,'-\vec{y}\,'\|} \, d^3y' \right) \Psi'(t',\vec{x}\,') = 0$$

# Schrödinger-Newton Equation (contd.)

$$C = \frac{2Gm^3a}{\hbar^2} \approx m^3[10^{10}\,u] \cdot a[5 \times 10^{-7}\,\mathrm{m}]$$

- Significant inhibition of spreading sets in around $C \approx 1$ (A. Großardt's talk).
- Ground state exists (E. Lieb 1977) with energy (Moroz-Penrose-Tod 1998)

$$E \approx \frac{G^2m^5}{\hbar^2} = C \cdot \frac{Gm^2}{2a} \approx mc^2 \cdot \left(\frac{m}{m_P}\right)^4$$

- Sanity check

$$\frac{GE}{c^4a} \approx \frac{G^4m^8}{\hbar^4c^4} = \left(\frac{m}{m_p}\right)^8 \ll 1$$

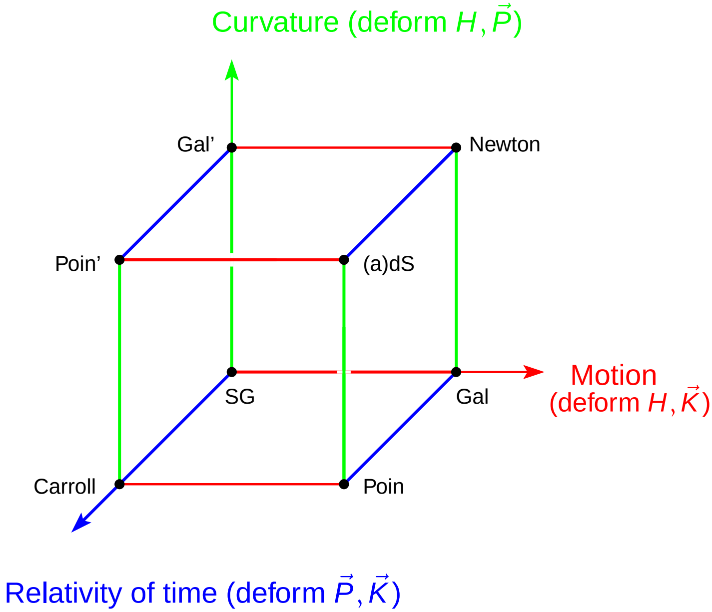
$$\Leftrightarrow m < m_p \approx 10^{19} \cdot u \qquad \text{(OK!)}$$

# Possible kinematics

- What (Lie-)groups could be automorphism groups of spacetime? Simple hypotheses would be (Bacry & Lévy-Leblond 1967)
  1. There are 10 generators:
  $$H\,, \quad \vec{P}\,, \quad \vec{J}\,, \quad \vec{K}\,.$$
  2. so that ($a, b, c$ cyclic)
  $$[J_a\,,\,H]=0\,,\quad [J_a\,,\,P_b]=P_c\,,\quad [J_a\,,\,J_b]=J_c\,,\quad [J_a\,,\,K_b]=K_c\,.$$
  3. and
  $$K_a \quad \text{are not compact}$$
  4. and
  $$\begin{aligned}\Pi:\quad H &\to \phantom{-}H\,,\quad \vec{P}\to -\vec{P}\,,\quad \vec{J}\to\vec{J}\,,\quad \vec{K}\to -\vec{K}\\ \Theta:\quad H &\to -H\,,\quad \vec{P}\to \phantom{-}\vec{P}\,,\quad \vec{J}\to\vec{J}\,,\quad \vec{K}\to -\vec{K}\end{aligned}$$

     are Lie-algebra automorphisms
- Then the group is one of the following:

# Possible kinematics from deformations



# New possible kinematics

- The variety of possible deformations clearly depends on the algebraic category in which you deform.
- New ideas about generalised kinematics take the universal enveloping algebra of a lie group, which is a spacial Hopf algebra, and deform in the category of Hopf algebras ($\rightarrow$ Quantum Groups).
- A similar analysis to that above can then be repeated. An early attempt is due to Bacry (1992), more recent ones are Gromov & Kuratov (2006).
- The interesting fact is that Quantum Groups may naturally appear as (generalised) symmetries in models of Quantum Gravity. Doubly Special Relativity and the $\kappa$-Poincaré algebra are special cases.
- A fascinating question concerns the associated geometries, i.e., a generalisation of Klein's “Erlanger Programm”.

# Very special relativity

- Usually breaking of LI are thought to arise by privileged observers:

  $$\longrightarrow \text{unit timelike vector field}$$

- An interesting alternative was pointed out by Cohen & Glashow (2006):

  $$\mathrm{SL}(2,\mathbb{C}) \rightarrow \text{upper triangular subgroup} \cong \mathrm{Sim}(2)$$

- Generators

  $$T_1 := K_x + J_y\,, \quad T_2 := K_y - J_x\,, \quad J_z\,, \quad K_z\,.$$

- **Defining property: Adjoining parity, or time reversal, generates $\mathrm{SL}(2,\mathbb{C})$.**

# VSR properties

- Translations are not broken ($\to \mathrm{ISim}(2) = \mathbb{R}^4 \rtimes \mathrm{Sim}(2)$)
- CPT invariant.
- $\mathrm{Sim}(2)$ acts transitively on standard mass shell.
- Compatible with current limits on VLI.
- No spurion fields exist for $\mathrm{Sim}(2)$.
- Not covered by existing test-theories (Mansouri-Sexl).
- Compatible with supersymmetry (Cohen & Freedman 2007).
- Can account for neutrino masses without violating lepton number or introducing sterile states (Cohen & Glashow 2006):

$$\left(\not{p} - \frac{m^2}{2} \cdot \frac{\not{n}}{p \cdot n}\right) \nu_L = 0\,.$$

# General VSR

- Deformations of

  $$\mathrm{ISim}(2) := \mathbb{R}^4 \rtimes \mathrm{Sim}(2)$$

  were classified by Gibbons *et al.* (2007).

- Modified dispersion relations result:

  $$p^2 = m^2(1-b^2)\left[\frac{n\cdot p}{m(1-b)}\right]^{2b/(1+b)}$$

  where $b$ is the deformation parameter

- These correspond to Finslerian geometry (of Bogoslovsky type)

  $$ds = (ds_{\mathrm{Mink}})^{(1-b)/2}\,(n_\alpha\, dx^\alpha)^b$$

# Conclusion

- The formulation of EEP in QM is possible but unambiguous.
- Quantum tests of UFF are now standard.
- Quantum tests of UGR are also possible but not yet achieved (despite claims to the contrary).
- Gravitational fields created by quantum matter in the laboratory are 5-7 orders of magnitude further away than presently communicated, but possibly not entirely outside reach (space missions).
- Space based future experiments will almost certainly allow significant tests of EEP in low energy regime.
- Modifications of Poincaré invariance at highest energies?
- Many important topics were not touched upon: cosmology, decoherence, specific predictions, observables, space-time structure, .....
- **Go to talks by Klaus Fredenhagen, Sabine Hossenfelder, Claus Kiefer, Renate Loll, Catherine Meusburger, Thomas Thiemann, Marco Zagermann, and others!!!**